

World currency exchange rate cross-correlations

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Abstract. World currency network constitutes one of the most complex structures that is associated with the contemporary civilization. On a way towards quantifying its characteristics we study the cross correlations in changes of the daily foreign exchange rates within the basket of 60 currencies in the period December 1998–May 2005. Such a dynamics turns out to predominantly involve one outstanding eigenvalue of the correlation matrix. The magnitude of this eigenvalue depends however crucially on which currency is used as a base currency for the remaining ones. Most prominent it looks from the perspective of a peripheral currency. This largest eigenvalue is seen to systematically decrease and thus the structure of correlations becomes more heterogeneous, when more significant currencies are used as reference. An extreme case in this later respect is the USD in the period considered. Besides providing further insight into subtle nature of complexity, these observations point to a formal procedure that in general can be used for practical purposes of measuring the relative currencies significance on various time horizons.

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The financial markets offer an arena to quantitatively view the most complex aspects of human activity on the global scale. In terms of volume the currency market represents the largest market as its daily transactions total trillions of dollars. The related dynamics of the Foreign Exchange (FX) market [1] involves interactions with virtually all information around the world and, in particular, with the price changes on all other markets. The exchange rates between different currencies can thus be well represented by stochastic variables. Within the finite time horizons any such rate is therefore likely to lead to a nonzero correlation with another [2–4]. Collection of the corresponding correlation coefficients can be used to form a correlation matrix and the question which emerges is what are the patterns that are encoded in such a matrix. For the stock market this issue is by now quite well explored in the physics literature [5–7]. For instance, it turns out common that the bulk of eigenspectrum of the corresponding correlation matrices is largely consistent with predictions of the random matrix theory (RMT). Typically only a small fraction of eigenvalues stay sizably apart from the bounds prescribed by the RMT. The largest eigenvalue is associated with the most collective market component which represents the common factor driving the stocks participating in that market. A few smaller eigenvalues correspond to similar effects but on the level of various distinct market sectors. The logic of the currency market is somewhat different. No analogous common factor can for instance be

always expected. On the other hand there exist currencies — like at present the USD [8] — that play a major role in the contemporary world trading system and, on the opposite pole, there are those whose global role is marginal. How such effects can be quantified remains an issue both of the fundamental as well as of practical importance.

Our study is based on the daily FX time series [9] of 60 world currencies (including platinum, gold and silver, the full list of currencies is displayed in Fig. 3), $x_i^{(a)}(t)$, from the period Dec. 1998–May 2005, where the value of i th currency is expressed in terms of the base currency a . The data were preprocessed in order to remove some numerical artifacts (we did this by removing day-to-day exchange rate jumps larger than 5σ , losing in this way not more than 0.3% of data points) and to synchronize the gaps related to non-trading days. For each currency we obtained a time series of 1657 data points.

As usually, the logarithmic daily returns are defined $G_i^{(a)}(t; \tau) = \ln x_i^{(a)}(t + \tau) - \ln x_i^{(a)}(t)$, where the return time τ is also called the time lag. For n currencies there are in principle $n \times (n - 1)$ exchange rates. Neglecting friction (transaction costs) half of them are simply related to their inverse counterparts

$$G_i^{(a)}(t; \tau) + G_a^{(i)}(t; \tau) = 0. \quad (1)$$

In addition, it can be shown that the remaining exchange rates are not independent. Due to the triangular arbitrage we have additional $(n-2)(n-1)/2$ independent constraints

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called the triangle rules

$$G_i^{(a)}(t; \tau) + G_b^{(i)}(t; \tau) + G_a^{(b)}(t; \tau) = 0. \quad (2)$$

In effect, we have $(n - 1)$ independent time series.

For the present exploratory purpose the above basket of currencies constitutes a reasonable compromise that represents the global currency network. It includes the convertible currencies, from major up to the less liquid ones, whose price is settled in spot trading, as well as the non-liquid currencies whose exchange rates are fixed by the central banks. Interestingly, from the daily time scale perspective the dynamics of fluctuations of all the corresponding rates within this basket is governed by a similar functional form. For instance, we have verified explicitly, as one quantitative test, that the daily return distributions for these rates turn out exponential with similar accuracy. This may — and at least partly even should — originate from the trading elements that make the triangle rules work, which unavoidably introduces links between all the existing currencies. We therefore find it optimal, also for statistical reasons, to initiate our analysis with all the 60 currencies taken together.

In what follows we use the correlation matrix formalism in order to globally study the mutual dependencies among changes of the exchange rates within this basket. Any of the n currencies can then be used as a base currency for the remaining ones. The correlation matrices are therefore constructed for each of those n base currencies separately. This results in n matrices of the size $(n - 1) \times (n - 1)$, $\mathbf{C}^{(a)} \equiv [C^{(a)}]_{ij}$, where $i, j = 1, \dots, n - 1$ and a denotes the base currency ($a = 1, \dots, n$). To construct the correlation matrix one defines the auxiliary matrix $\mathbf{M}^{(a)}$ by normalized returns, $g_i^{(a)}(t; \tau) = [G_i^{(a)}(t; \tau) - \langle G_i^{(a)}(t; \tau) \rangle_T] / \sigma(G_i^{(a)})$, where $\sigma(G)$ denotes standard deviation of G . Taking $(n - 1)$ time series $\{g_i^{(a)}(t), g_i^{(a)}(t + \tau), \dots, g_i^{(a)}(t + (T - 1)\tau)\}$ of length T we can build an $(n - 1) \times T$ rectangular matrix $\mathbf{M}^{(a)}$. Finally, the symmetric correlation matrix is defined by

$$\mathbf{C}^{(a)} \equiv [C^{(a)}]_{ij} = \frac{1}{T} \mathbf{M}^{(a)} \widetilde{\mathbf{M}}^{(a)}, \quad (3)$$

where tilde means the matrix transposition. Each correlation matrix is determined by its eigenvalues ($\lambda_i^{(a)}$) and eigenvectors ($v_{ij}^a \equiv \mathbf{v}_i^a$), $\mathbf{C}^{(a)} \mathbf{v}_i^a = \lambda_i^{(a)} \mathbf{v}_i^a$. The eigenvalues are enumerated in the monotonic order, $0 \leq \lambda_{(1)}^{(a)} \leq \lambda_{(2)}^{(a)} \leq \dots \leq \lambda_{(n-1)}^{(a)}$. The trace of the matrix is always $\text{Tr } \mathbf{C}^{(a)} = n - 1$, i.e. equal to the number of time series.

In the present study each such matrix is dealt with separately and the main focus is put on identification of a potential non-random component. In general, as compared to a pure random case where the entries of the correlation matrix are distributed according to a Gaussian centered at zero, the real correlations may correspond either to the uniform displacement of such a distribution [10] or to the appearance of fatter tails [11]. Both such mechanisms generate an effective reduction of the rank of a leading com-

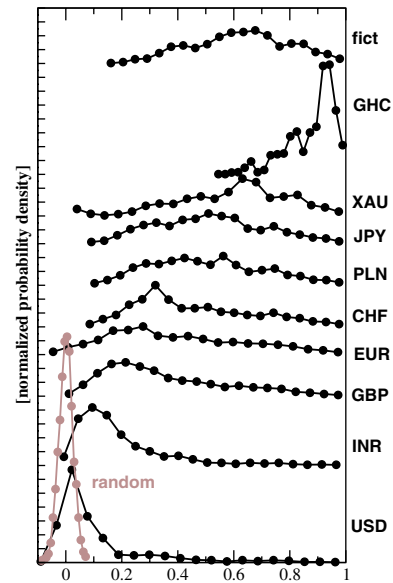


Fig. 1. Distribution of the off diagonal elements of correlation matrices, $C_{ij}^{(a)}$, $i > j$, for selected base currencies (a). For the sake of clarity each curve is appropriately shifted upwards, with vertical scale preserved. The gray curve corresponds to random (white noise) time series of returns. The subsequent plots are shifted up by integer number for better readability. One tick on the vertical axis corresponds to probability density difference equal 1.0.

ponent of the matrix under consideration and, as a consequence, a large eigenvalue as compared to the remaining ones is expected to be seen. In our case of $n = 60$ correlation matrices the distribution of their off diagonal entries strongly depends on which currency is used as the base currency. Several representative cases, including the most extreme ones, are shown in Figure 1.

First of all, they all differ from a Gaussian centered at zero. At first glance somewhat surprisingly however it is the case of the USD used as the reference currency that differs least from a Gaussian. The shape of the distribution in this case indicates that changes among the currencies expressed in the USD are both positively as well as negatively correlated. However, there is a visible asymmetry towards positive correlations, some of them being close to unity. As we move up in Figure 1, i.e., the other listed currencies are used as reference, the FX dynamics becomes more and more positively correlated. Both the maximum of the distribution as well as its mean systematically move to the larger positive values. In our basket of currencies an extreme situation one finds when all the currencies are expressed in terms of the GHC (Ghanian cedi). In this case almost all pairs of exchange rates have correlation coefficients larger than 0.7 and the maximum of their distribution is located at around 0.9. Finally, two ‘null hypotheses’ are generated and the corresponding distributions of the correlation coefficients also shown in Figure 1. One of them, termed fictitious (fict), is generated in such a way that the USD/fict exchange rate time series is represented by a sequence of uncorrelated random

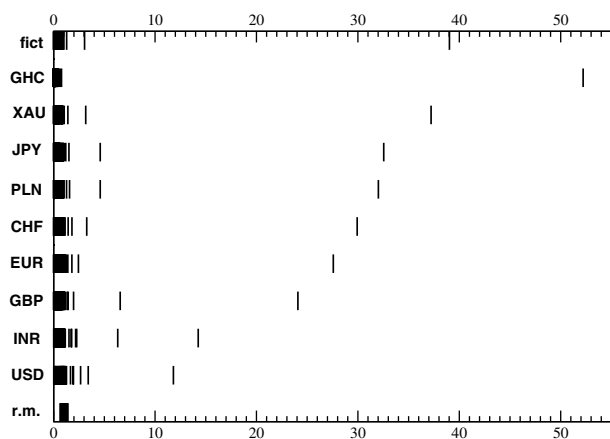


Fig. 2. Eigenspectra ($\lambda_i^{(a)}$) of correlation matrices for the base currencies as in Figure 1. The random matrix eigenspectrum is also displayed for comparison and denoted as r.m.

numbers. The other real currencies are expressed in terms of such currency by using the relation: $\text{currency}/\text{fict} = \text{currency}/\text{USD} * \text{USD}/\text{fict}$. By construction, the fictitious currency is entirely disconnected from the real world economy. Somewhat unexpectedly the resulting distribution of the correlation coefficients does not differ much from large fraction of cases when the real currencies are used as the reference. In the second (random) case all the time series are replaced by the sequences of pure uncorrelated random numbers of the same length as the original ones. Clearly, this results in the centered at zero Gaussian distribution of matrix elements.

The most straightforward and compact characteristics of the correlation matrix is its eigenspectrum. Such full eigenspectra for the cases presented in Figure 1 are shown in Figure 2. From our present perspective the most interesting quantity is magnitude of the largest eigenvalue, and especially a gap it develops, because it may reflect an amount of non-random correlations. The relative locations of these largest eigenvalues can easily be seen to go in parallel with the differences in distributions of the corresponding matrix elements discussed in Figure 1. Consistently, for the USD we find the smallest value while for the GHC the largest. In the later case the correlations are so strong that the largest eigenvalue exhausts about 85% of the total matrix trace enslaving [12] the other eigenvalues of this correlation matrix to the region close to zero. Several intermediate cases, especially when the gold (XAU) is used as the base currency, do not differ much from the above described case of a fictitious currency. The number of eigenvalues is here too small to perform a fully quantitative evaluation of the noise content in the bulk of eigenspectrum by relating our results to the predictions of the RMT. In the low lying part of the spectra there is some overlap with the case described above as random but it seems unlikely that the situation is as extreme as for the stock market, even after correcting for the repelling effect of the largest eigenvalue. This may originate from the tendency of currencies to exhibit correlated clusters at several levels of their interactions [13, 14]. The presence of

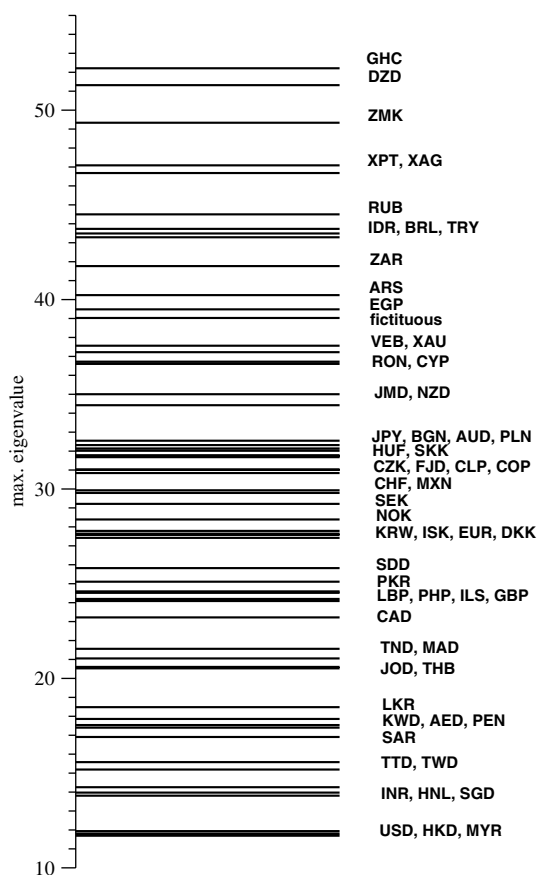


Fig. 3. Maximal eigenvalues ($\lambda_{59}^{(a)}$) for correlation matrices with all 60 considered currencies taken as the base currency (index a), including the fictitious (random) currency.

several (typically 6–8) eigenvalues larger than the RMT upper bound can be considered as a manifestation of such effects.

Iterating the above procedure for all the currencies selectively used as a base currency one can draw the whole ladder of the corresponding largest eigenvalues. The collection of all such eigenvalues for our case of $n = 60$ currencies, plus the case that above is termed a fictitious currency, is presented in Figure 3. The two cases more in detail discussed already above — of the USD and of the GHC — constitute the lower and the upper bound correspondingly in this ladder. Several effects can be discussed based on such a representation of the currencies. The one that is especially worth drawing attention is that it opens room for assigning a relative significance in the world economy to any particular currency. Indeed, it is the USD that is most frequently used in the world trading system and can be considered the world most influential currency in the period studied here. At the same time the exchange rate cross-correlations of the other currencies expressed in terms of the USD do involve a large eigenvalue which is separated from the rest of eigenspectrum by a sizable gap. The currencies considered do correlate even more (larger gap) when expressed in terms of any other selected currency. This in particular applies to the fictitious

currency which in the ladder of Figure 3 is located relatively high, but still in the region of many real currencies. A number of them is represented by even significantly larger eigenvalues. As it can be inferred already from Figure 1, the increase of the largest eigenvalue is accompanied by the systematically increasing delocalization — almost towards uniformity in case of GHC — of the corresponding eigenvector components. The magnitude of this largest eigenvalue can thus be associated with the degree of collectivity.

A perspective to understand the above results is that the world currency network does not pay much attention to changes in value of a peripheral currency. Majority of the currency exchange rates expressed in that particular currency then synchronously adjust in the same direction which introduces a large reduction of an effective dimensionality of the corresponding correlation matrix which results in one very large eigenvalue [15]. The eigenvalues even larger than a peripheral currency representative, i.e. the fictitious currency eigenvalue, correspond to those currencies that experience a violent depreciation (potentially also appreciation) and all the other currencies uniformly synchronize in reflecting this effect. On the other hand, changes of value of a significant currency — due to its links to the fundamentals of the world economy — may cause a rich diversity of reactions of the other currencies which in the present formal approach is seen as a sizable degradation of collectivity. The dynamics in this case is definitely more complex and the gap between the largest eigenvalue and the rest of the spectra — though still pronounced — is smaller than in any other case considered, except of course for a somewhat trivial, artificial, totally random case that develops no gap at all. This observation provides further arguments in favor of characteristics that real complexity exhibits when formulated by means of matrices [15].

A note of caution is of course needed as far as blind direct practical conclusions are to be drawn in the opposite direction. This appealing scheme, in which the world's most significant currencies used as a base tend to develop relatively small non-random component while the least influential currencies correspond to a larger collectivity, is blurred in Figure 3 due to possible strong economic ties between different countries or explicit pegs fixing exchange rates of some currencies in respect to reference ones. The exchange rates of a pegged currency follow the exchange rates of its reference and this obviously implies similarity of the eigenspectra of the corresponding correlation matrices. For example, the largest eigenvalue for USD is in Figure 3 accompanied by the largest eigenvalues for MYR and HKD. In fact, significance of neither of the two latter currencies can be compared to the significance of American dollar. The coinciding eigenvalues are in this case a result of the artificially stabilized HKD/USD rate due to the “linked exchange rate” mechanism in Hong Kong and the MYR/USD peg introduced by the Malaysian central bank in response to the Asian crisis of 1997. Also in the case of strong economic ties between two countries their currencies are expected to behave in a similar way [2],

as it is, for instance, with some Latin American currencies whose values fluctuate according to the fluctuations of USD. This effect can also lead to a decrease of the largest eigenvalue for the currencies being satellites of the world's leading ones like USD or EUR. It should also be noted that the analysis based solely on the time series of daily returns inevitably neglect such an important economic factor crucial for the currency stability as the inflation.

The above reasons indicate that the relation between the currency importance and its position in the ladder of Figure 3 is an implication rather than an equivalence. Although the relative currency significance affect the magnitude of the associated largest eigenvalue in such a way that the more significant currency the smaller the eigenvalue gap, the position occupied by a given currency cannot decisively determine its significance without some additional knowledge of the country's economy and the monetary policy of its central bank. Nevertheless, a representation of currencies in the spirit as is illustrated in Figure 3 seems to also indicate a practically useful framework to quantitatively assign the relative significance of currencies selected from their basket at various time horizons. As a further test it is instructive to make an analogous analysis in some distinct currencies sectors independently. We did it for instance for tradeable as well as for non-tradeable currencies separately and found that the relative locations of the largest eigenvalues remain similar. It thus seems that on daily time scale the mechanism of correlations has some common elements for all the currencies. This may not be true on the shorter scales however.

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